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LETTER TO THE EDITOR

A new universality class for the metal–insulator transition problem

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Abstract. A new scenario is proposed for a class of disordered interacting electronic systems undergoing a metal–insulator transition. The scenario applies in the absence of spin-flip mechanisms to systems that are not nearly ferromagnetic. At the phase transition both the spin susceptibility and the coefficient of the linear term in the specific heat diverge with identical exponents, and the electrical conductivity vanishes continuously with a different exponent. Approximate values are given for the critical exponents of transport and thermodynamic properties in three-dimensional systems.

The metal–insulator transition that occurs in systems without spin-flip mechanisms has proven to be very difficult to describe [1]. The reason has been that in low-order renormalization group (RG) calculations the triplet interaction amplitude, k_t, scales toward strong coupling much faster than the singlet interaction amplitude, k_s. As a result, $\gamma_t = |k_t/k_s|$ diverges at a finite scale, indicating that perturbative RG methods cannot be used to describe the system. The physical reason for this had been speculated to be some kind of spin instability [2], which occurs before or simultaneously with the metal–insulator transition (MIT).

Recently some progress has been made towards resolving this problem [3, 4]. It was shown in [3] that apart from the long-searched-for MIT there is a second zero-temperature phase transition in the system, which leads from a Fermi liquid (FL) to an incompletely frozen spin (IFS) phase, the latter being a charge conductor. This new phase transition was related to the runaway flow encountered in the earlier RG calculations and could be described by means of an infinite order resummation of perturbation theory. Kirkpatrick and Belitz [3] found that in three dimensions (3D) the FL–IFS phase transition precedes the MIT if the Fermi-liquid parameter F_0^a is close to -1. For smaller values of $|F_0^a|$ we found that γ_t does *not* diverge, even though lowest order calculations suggest otherwise. This suggests the phase diagram for 3D disordered interacting electrons shown in figure 1. Here I denotes a charge insulator phase and g is a dimensionless measure of disorder. For small values of $|F_0^a|$ it is natural to assume a direct FL–IFS transition, since with γ_t staying finite there should be no competing instability. Kirkpatrick and Belitz [3] were unable to describe this MIT since, amongst other obstacles, they assumed that disorder behaves trivially under renormalization. This is true only if $\gamma_t \to \infty$.

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Very recently this picture has been confirmed and expanded [4]. The results for our present purposes are as follows. (i) For dimensionalities $d > d_c > 2$ there always exists a multicritical point, M, in the phase diagram. Below this point, γ_t remains finite under scaling, and no IFS phase exists. (ii) For $d \to d_c$, M approaches the axis $F_0^a = 0$, and for $2 < d < d_c$, the IFS phase *always* separates the FL from the I phase.

For a description of the FL–I transition (the line M–C in figure 1) these observations have important consequences. First of all, since this MIT does not exist in a non-vanishing neighbourhood of $d = 2$, it cannot be described by the customary $\epsilon = d - 2$ expansion. This breakdown of the ϵ-expansion is one of the reasons why the MIT has so far eluded all attempts at theoretical description. Short of finding an upper critical dimensionality for the problem (which has not been possible even for the corresponding non-interacting problem) it is impossible to obtain a 'controlled' theory in the sense of an ϵ-expansion. Furthermore, we have learned that γ_t does not necessarily scale towards infinity, even if the flow equations have been derived in the limit $\gamma_t \gg 1$. It is then natural to assume that the FL–I transition will correspond to a fixed point (FP) where both g and γ_t approach finite values g^* and γ_t^*, respectively.

In this paper we show that such a FP is indeed present in the two-loop RG equations for $d = 3$. This FP corresponds to the MIT represented by line M–C in figure 1. Since, for the reasons given above, our description necessarily lacks a small parameter, it is, strictly speaking, a scenario rather than a theory of the MIT. Nevertheless it is the first description of a MIT for interacting electrons without spin-flip mechanisms that is consistent with all known properties of the system derived within well controlled frameworks.

We start with the two-loop RG equations derived in [5],

$$dg/dx = -\epsilon g + (g^2/2)[5 - 3(1 + 1/\gamma_t)\log(1 + \gamma_t)] \tag{1a}$$

$$d\gamma_t/dx = (g/4)(1 + \gamma_t)^2 - \tfrac{3}{8}g^2\gamma_t^3[5/2 - \log(1 + \gamma_t)] \tag{1b}$$

$$dh/dx = (h/4)g(3\gamma_t - 1) + \tfrac{3}{4}hg^2\gamma_t^2. \tag{1c}$$

For $\epsilon = d - 2 > 0$, (1) contains the first two non-vanishing contributions to the flow equations for g, γ_t and h. The one-loop terms in (1) are exact; the two-loop terms were derived in the $\gamma_t \gg 1$ limit. Here we will use them for $\gamma_t \sim 0(1)$ (and $g \sim 0(1)$). The justification for the former is that if γ_t does not scale to infinity it must necessarily stay finite. If we find a FP with a finite γ_t^*, this will therefore be consistent with our previous integral equation approach in $d = 3$ [3,4]. The latter extension is necessary since in the region close to $d = 2$, where g^* would be small, the transition is absent, cf above. In (1), $x = \log\ b$ with b the RG length rescaling factor. Equation (1) is valid for electrons interacting via screened Coulomb interactions and in (1*a*) the pure localization correction has been taken into account [1]. Equation (1*c*) shows the relevance of a finite frequency or temperature as a zero-temperature phase transition is approached.

The numerical solution of (1*a*, *b*), figure 2, shows that there are several FPs as a function of ϵ and initial values of g and γ_t. The physical FP, if any, is the one that first occurs with increasing disorder. For $\epsilon = 0$ (figure 2(*a*)) and $g_0 = g(b = 1)$ very small, g first increases and then decreases and γ_t diverges at a finite $x = \log\ b$ [1]. The ground state in $d = 2$ is therefore not a Fermi liquid. For larger g_0 one finds a stable FP $(g^*, \gamma_t^*) = (0.51, 2.08)$ with an associated separatrix separating regions in parameter space when γ_t diverges at a finite scale from those where g

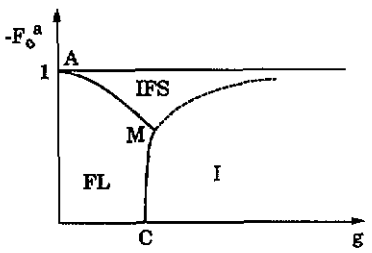

Figure 1. Schematic phase diagram for a three-dimensional disordered Fermi liquid. g is a dimensionless measure of disorder, and F_0^a is the usual Fermi liquid parameter. The three phases shown are a Fermi liquid (FL), an incompletely frozen spin (IFS), and an insulator (I) phase, respectively. No information is available at present about the IFS–I phase transition curve (broken curve). In particular we do not know if the broken curve will reach the line $|F_0^a| = 1$ at a finite disorder. A and C denote end points of transition lines, and M is a multi-critical point.

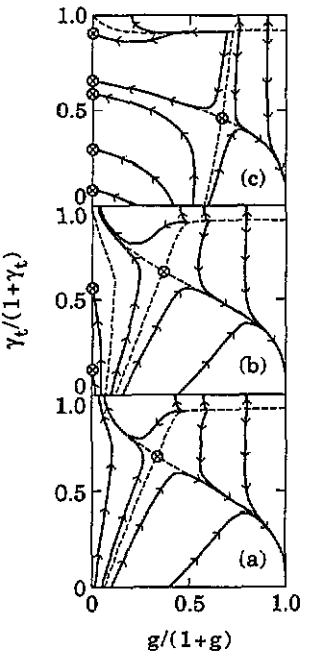

Figure 2. Flow diagram showing the solution of (1*a*, *b*) for three different dimensionalities $d = 2 + \epsilon$: (*a*) $\epsilon = 0$, (*b*) $\epsilon = 0.05$, (*c*) $\epsilon = 1$. Fixed points are denoted by crossed circles, and separatrices by broken curves.

diverges at a finite scale. Equation (1) is only capable of describing how metallic (small g) behaviour breaks down. For $\epsilon = 0$, this breakdown occurs already at arbitrarily small g_0 by means of $\gamma_t \to \infty$. The FP is therefore inaccessible and has no physical significance. For $0 < \epsilon \ll 1$ (figure 2(*b*)) and small g_0, a Fermi liquid FP ($g^* = 0, \gamma_t^* =$ finite number) is reached. With increasing g_0, one enters a region when again γ_t diverges at a finite scale. The separatrix between these two regions can be related to the FL–IFS phase transition [3, 4]. Since it involves runaway trajectories, this transition requires an infinite resummation of the loop expansion and cannot be described by (1); this has been discussed in detail elsewhere [3, 4]. For still larger g_0, there is again an inacccessible stable FP.

For $\epsilon = 0(1)$, the behaviour of the flow changes again qualitatively, cf figure 2(*c*). For small g_0 (and a γ_t^0 that is not too large) there always is a Fermi liquid FP. With increasing g_0 one reaches the stable FP separating $g \to \infty$ flow with no $\gamma_t \to \infty$ trajectories in between. The FP is therefore accessible and describes an MIT (line M–C in figure 1). For $\gamma_t^0 \gg 1$ there is again a separatrix related to the FL–IFS phase transition (line M–A in figure 1), confirming the existence of the multi-critical point M in figure 1.

We conclude that for ϵ larger than a critical value $0 < \epsilon_c < 1$ there is an accessible, stable FP describing an MIT. Because there are no runaway trajectories at this FP, we do not have to perform an infinite resummation as for the pseudo-magnetic transition but rather we can use (1) to approximately describe it. Furthermore, we know that the FP is unlikely to be an artifact of the two-loop RG equations because of our previous work [3,4] which shows that γ_t must scale to finite value for these parameters.

The MIT FP predicted by (1) is defined by the equations,

$$g^* = 2\epsilon/[5 - 3(1 + 1/\gamma_t^*)\log(1 + \gamma_t^*)] \tag{2a}$$

$$5 - 3(1 + 1/\gamma_t^*)\log(1 + \gamma_t^*) = \tfrac{15}{2}[\epsilon\gamma_t^{*3}/(1 + \gamma_t^*)^2][1 - \tfrac{2}{5}\log(1 + \gamma_t^*)]. \tag{2b}$$

Numerically solving (2) gives g^* and γ_t^*. In three dimensions, $g^* \simeq 2.01$ and $\gamma_t^* \simeq 0.84$. The linearized RG eigenvalues which give the critical exponents for the MIT are determined by expanding (1) around $(g^*, \gamma_t^*, h^* = 0)$. Equations (1*a*) and (1*b*) give one relevant eigenvalue, $\lambda_+ = 1/\nu$, related to the correlation length exponent, ν, and one irrelevant eigenvalue λ_-. This shows that our FP is indeed stable, cf also figure 2. The linearization of the right-hand side of (1*c*) gives $h\kappa/\nu$ (this defines the exponent κ) and the dynamical scaling exponent is $z = d + \kappa/\nu$. For $d = 3$ the predicted exponents are

$$\nu = 1/\lambda_+ = 0.75 \qquad \lambda_- = -4.08 \qquad z = 5.91. \tag{3}$$

Near this MIT the electrical conductivity, σ, the spin susceptibility, χ_s, and the coefficient of the linear term in the specific heat, $\gamma = \lim_{T<1} C/T$, satisfy the scaling laws [6,7],

$$\sigma(t, T) = b^{-\epsilon} f_\sigma(b^{1/\nu}t, b^z T) \tag{4a}$$

$$\chi_s(t, T) = b^{\kappa/\nu} f_\chi(b^{1/\nu}t, b^z T) \tag{4b}$$

$$\gamma(t, T) = b^{\kappa/\nu} f_\gamma(b^{1/\nu}t, b^z T). \tag{4c}$$

In (4), t is the dimensionless distance from the MIT at $T = 0$ and the f_a are scaling functions. An approximate expression for the line M-C in figure 1 can be determined by computing the RG critical surface for our FP. In $d = 3$ we find

$$g_0 = 1.69 + 0.39\gamma_{t0}. \tag{5}$$

In three dimensions (4) predicts, for example,

$$\sigma(t \to 0, T = 0) \sim t^{0.75} \tag{6a}$$

$$\chi_s(t = 0, T \to 0) \sim T^{-0.49} \tag{6b}$$

$$\gamma(t = 0, T \to 0) \sim T^{-0.49}. \tag{6c}$$

We discuss our results in the form of some remarks:

(i) Previously [2–4] the $\gamma_t^* \to \infty$ FP or phase transition has been discussed in considerable detail. In this paper we discuss the only other non-trivial possibility: $\gamma_t \to \gamma_t^* =$ finite constant. That this possibility can occur is consistent with our previous work [3,4].

(ii) The location of the multi-critical point M in figure 1 is of considerable interest from an experimental point of view. In general this is a non-universal quantity which is difficult to determine. Our previous integral equation approach suggested that the critical F_0^a was close to -0.7. This value, however, proved to be strongly cut-off dependent and a reliable determination is at present not possible.

(iii) We stress that the values of the exponents given by (3) and (6) are only approximate, since we are applying an expansion in g to the regime $g = 0(1)$. Moreover, the solution of (2) shows that the critical disorder, g^*, does not approach zero for $\epsilon \to 0$. The reason is that our FP loses its physical meaning for ϵ smaller than some $\epsilon_c > 0$, as pointed out above.

(iv) Note that (3) satisfies the rigorous inequality [8] $\nu \geqslant 2/3 (= 2/d$ in d dimensions). Also note that this inequality and (4*a*) require that the zero-temperature conductivity exponent must satisfy $s \geqslant 2/3 (s \geqslant 2(d-2)/d)$.

(v) The multi-critical point M separates the pseudomagnetic behaviour resembling enhanced paramagnon of [2,3] from a behaviour of the highly correlated electrons that is more reminiscent of a Brinkman–Rice picture [9]. For large γ_{t0}, the former is realized. The renormalized triplet interaction amplitude, k_t, increases strongly with increasing disorder, while the singlet interaction amplitude k_s is only weakly affected [2–4]. For small γ_{t0}, the latter is realized, and both k_t and k_s increase and diverge in identical ways across the line M–C in figure 1. It is physically reasonable that this change in behaviour is controlled by the Fermi-liquid parameter F_0^a.

(vi) Our results bear some resemblance with the local-moment two-fluid picture that has been used [10] to describe doped semiconductors near their MIT. In both cases the spin susceptibility and the linear term in the specific heat diverge with identical exponents as $T \to 0$. On the other hand, an important difference is that the local-moment picture predicts that these quantities also diverge, as $T \to 0$, in the metallic phase. This suggests a straightforward experimental check to distinguish between the two theories: according to our analysis both $\chi_s(T \to 0)$ and $\gamma(T \to 0)$ should saturate in the metallic phase while in the local moment picture they diverge. It is possible, however, that in the local-moment picture the divergence will take the form of a very weak singularity [11] and experimentally the two scenarios may well be indistinguishable at realizable temperatures.

(vii) The best studied system undergoing a MIT is Si:P with a F_0^a that can be estimated to be of order -0.5. If the estimate of the multi-critical point in figure 1 given in point (ii) above is accurate then the present theory, rather than the IFS transition theory, should be relevant for this material. Precision measurements [12] of the conductivity in the mK temperature range give a critical exponent close to 0.5. As mentioned in point (iv) above, this seems to violate a rigorous inequality. Specific heat measurements [13] deep in the insulator suggest a temperature scale in the insulating phase that is less than 1 mK. If this anomalously low scale persists into the metallic phase then the experiments are not measuring asymptotic critical exponents but rather some effective exponents. This also suggests that the critical regime in this system may be immeasurably close to zero temperature. We note in passing that the critical exponents given by (6*b*) and (6*c*) are consistent with experiments [10, 13, 14] which yielded exponents close to 0.5 or 0.6.

Apart from this experimental evidence, we also have theoretical evidence that the true asymptotic zero-temperature exponents would be observable only at extremely low temperatures. From (4) it follows that the temperture is effectively low if $T^* \ll$

$t^{\nu z}$. Here $T^* = T/T_0$ is a dimensionless temperature with $T_0 \leqslant T_F$. For Si:P, $T_F \simeq 10^2$ K. This and (3) give $T \ll 10^2 t^{4.43}$ K. Close to the critical point ($T < 10^{-1}$) this temperature range is much smaller than the one currently accessible by experiment [12]. The general point is that a large dynamical scaling exponent implies that the zero-temperature behaviour sets in only at very low temperatures. If one accepts the validity of the scaling laws, (4), it then follows from the experimental observation $\chi_s(T) \sim T^{-0.5}$ that z must be close to $2d = 6$. We can thus argue that the experimentally observed behaviour of $\chi_s(T)$ (and $\gamma(T)$) already tells us that the experiments measuring $\sigma(t)$ were not in the asymptotic low-temperature regime. The same scaling argument also predicts that a large Fermi temperature makes the low-temperature regime more accessible.

Finally, for $F_0^a = -0.5$, a numerical solution of (1) predicts that on the metallic side the Wilson ratio W ($W = \chi_s\gamma_0/\gamma\chi_{s0}$, with χ_{s0} and γ_0 the degenerate electron gas values for χ_s and γ) decreases slightly with temperature before increasing to about three. The decrease may be an artifact of the approximate RG equations: minor changes in (1) lead to a monotonically increasing Wilson ratio with decreasing temperature and to larger FP values. Generically, the FP value of γ_t (which is proportional to W) at the MIT, $\gamma_t^{*,\mathrm{MIT}}$, can be greater than or less than the metallic or Fermi liquid FP value, $\gamma_t^{*,\mathrm{FL}}$. As a function of decreasing temperature, γ_t will have a plateau or valley (if $\gamma_t^{*,\mathrm{MIT}}, < \gamma_t^{*,\mathrm{FL}}$) or peak (otherwise) near $\gamma_t^{*,\mathrm{MIT}}$ as the MIT is approached. In principle this predicted behaviour can be probed experimentally by systematically considering systems with different values of γ_{t0}.

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